DESIGN OF RESISTANCE N-PORT NETWORKS WITH DEPENDENT SOURCES

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THESIS

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ABSTRACT

The classical n-port resistance network design problem is defined and a common approach to its solution based on parameter optimization is offered. An alternative approach is then proposed in which dependent sources are introduced into the network and where dependent source parameters assume the primary roles in the parameter optimization problem. Development of the alternative approach leads to specification of a related adjoint network and ultimately to the generation of an iterative design algorithm.

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I. INTRODUCTION

A. CLASSICAL N-PORT DESIGN

Design of the n-port resistance network is a classical problem in circuit theory [1]. Necessary and sufficient conditions are sought which guarantee realizability of a given real, symmetric, nonnegative definite nxn matrix as an admittance or an impedance matrix of an n-port network. The network is to consist entirely of real, positive-valued, two-terminal resistance elements. Such necessary and sufficient conditions do not exist, in general.

Solution of the problem using parameter optimization theory with the aid of the digital computer has been shown to be a sound practical approach [2]. The parameters to be optimized are simply the individual branch resistances of the n-port network. In this context, optimizing parameters implies adjusting the branch resistance (conductance) values of the n-port network to achieve the desired network specified by the given impedance (admittance) matrix. In lieu of the set of necessary and sufficient conditions originally sought, application of parameter optimization theory and the subsequent generation of an iterative design algorithm result in convergence of the algorithm for realizable matrices and no convergence for unrealizable matrices.

B. AN ALTERNATIVE APPROACH

An alternative method is now proposed for solution of the parameter optimization problem where individual branch resistances are held constant and dependent sources are introduced into the network. The four basic types of dependent sources are shown in Fig. 1.1. The parameters of the dependent sources now assume the primary roles in the optimization problem. In addition, introduction of the dependent sources implies that more than one branch may now connect a pair of nodes. Whereas the original design algorithm relied on the single parameter influence of branch resistance variation, the proposed alternative method allows multi-parameter influence in the variation of each of the dependent source parameters.

Consistent with the original formulation, an exact solution to the resistance n-port design problem safely may be assumed to be impossible, in general. Therefore, a quadratic error criterion is employed to define the parameter optimization problem. Specifically, the weighted sum of the squared errors between actual and desired entries in the given, nxn matrix at least assures convergence to a local minimum for realizable matrices. A typical quadratic performance criterion would be

$$J(\underline{p}) = 1/2 \sum_{\ell=1}^{n} \sum_{k=1}^{n} w_{k\ell}(y_{k\ell}-\hat{y}_{k\ell})^2 \qquad (1\text{-}1)$$

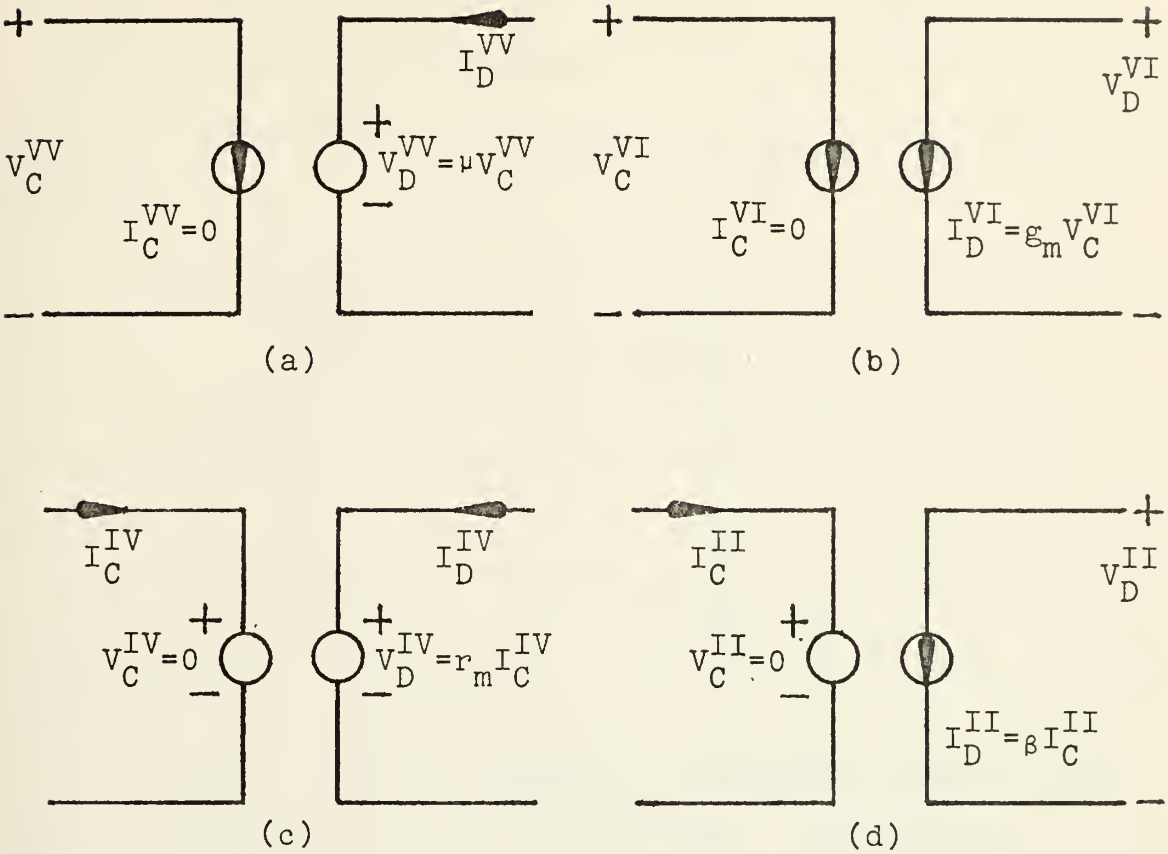

Fig. 1.1. Assumed configurations for the four basic types of dependent sources.

(a). Voltage-controlled voltage sources. Superscript VV identifies branches of these sources.

(b). Voltage-controlled current sources. Superscript VI identifies branches of these sources.

(c). Current-controlled voltage sources. Superscript IV identifies branches of these sources.

(d). Current-controlled current sources. Superscript II identifies branches of these sources.

The C subscript identifies the controlling branch and the D subscript identifies the dependent branch.

where $\underline{p}$ represnets a general vector of parameters to be optimized for an n-port network, and $\hat{y}_{k\ell}$ represents the desired value of the $k\ell^{th}$ entry of a general nxn admittance matrix

$$\underline{Y} = \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1n} \\ y_{21} & \cdots & \cdots & y_{2n} \\ \cdot & & & \\ \cdot & & & \\ \cdot & & & \\ y_{n1} & \cdots & \cdots & y_{nn} \end{bmatrix}$$

whose $k\ell^{th}$ entry is $y_{k\ell}$.

The $w_{k\ell}$ are real, nonnegative weighting factors which add design flexibility to the problem. For example, a given entry in the nxn admittance or impedance matrix may be ignored by choosing the corresponding $w_{k\ell}$ weighting factor to be zero. In addition for situations in which the given matrix is nonrealizable, appropriate selection of the weighting factors is equivalent to effecting engineering compromises in the design. Finally, when dealing with an iterative design scheme, adjustment of the weighting factors during the course of iteration may serve to speed convergence.

II. FORMULATION OF THE PROBLEM

A. N-PORT DESIGN AS A SIGNAL APPROXIMATION PROBLEM

The problem formulation parallels that of Director [3] with modifications for the introduction of dependent sources. Only the realization of a given, real, nxn admittance matrix is considered, recognizing the close similarity of the impedance realization problem.

The resistance n-port design problem with dependent sources is treated as a signal approximation problem. As shown in Fig. 2.1, the general network under consideration, denoted as $\mathcal{N}$, is assumed to have voltage excitation and current response. Since the problem deals strictly with linear time-invariant resistance networks with the dependent sources, consideration need be given only to n linearly independent n-vector dc port voltage excitation signals and their n corresponding n-vector dc current response signals. The n-port resistance network with dependent sources is to be characterized by a given, real, constant, nxn admittance matrix $\underline{Y}$ requiring that the dc independent voltage source port excitation n-vector

$$\underline{v} = (v_1\ v_2\ .\ .\ .\ v_n)^T$$

and the dc port current response n-vector

$$\underline{i} = (i_1\ i_2\ .\ .\ .\ i_n)^T$$

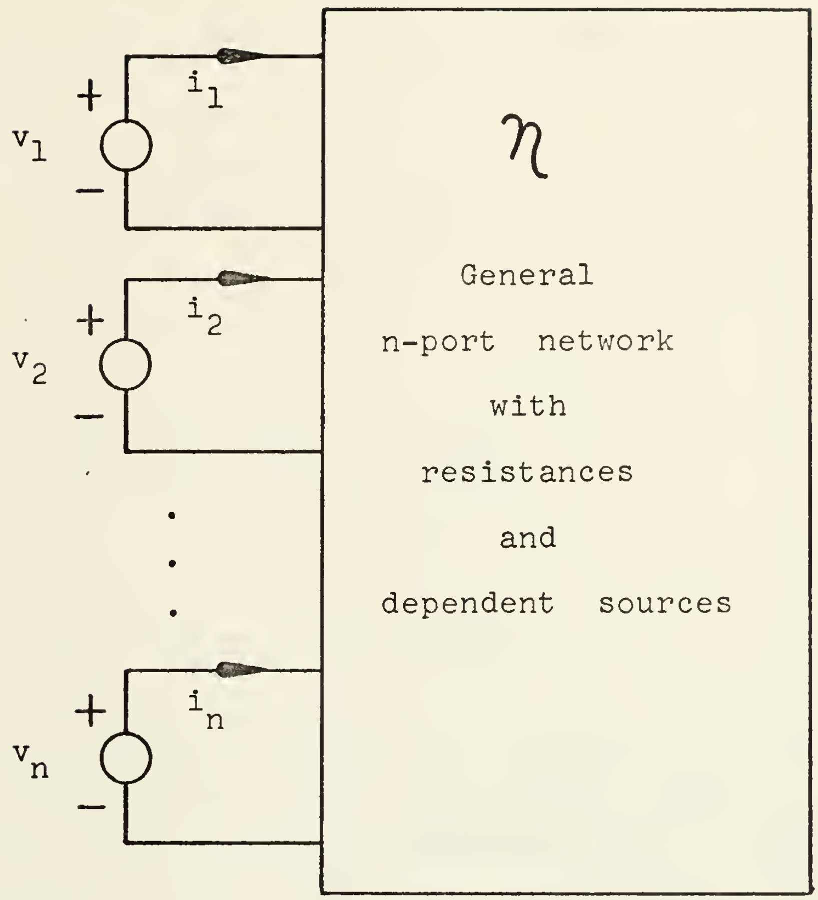

Fig. 2.1. General n-port network.

be related by

$$\underline{i} = \underline{Yv} \; . \tag{2-1}$$

In addition, define the nxn nonsingular voltage excitation matrix

$$\underline{V} = [\; \underline{v}_1 | \underline{v}_2 | \; . \; . \; . | \underline{v}_n]$$

where

$$\underline{v}_1 = (v_{11} \; v_{21} \; . \; . \; . \; v_{n1})^T$$
$$\underline{v}_2 = (v_{12} \; v_{22} \; . \; . \; . \; v_{n2})^T$$
$$\vdots$$
$$\underline{v}_n = (v_{1n} \; v_{2n} \; . \; . \; . \; v_{nn})^T \; .$$

The n columns of $\underline{V}$, having been denoted as $\underline{v}_j$ $(j = 1,2,...,n)$, represent a set of linearly independent port voltage excitation signals. They define the corresponding nxn port current response matrix

$$\underline{I} = [\; \underline{i}_1 | \underline{i}_2 | \; . \; . \; . | \; \underline{i}_n]$$

where

$$\underline{i}_1 = (i_{11} \; i_{21} \; . \; . \; . \; i_{n1})^T$$
$$\underline{i}_2 = (i_{12} \; i_{22} \; . \; . \; . \; i_{n2})^T$$
$$\vdots$$
$$\underline{i}_n = (i_{1n} \; i_{2n} \; . \; . \; . \; i_{nn})^T \; .$$

The more general form of equation (2-1) can now be written

$$\underline{I} = \underline{YV} \tag{2-2}$$

Since $\underline{V}$ is nonsingular (because its columns are chosen to be linearly independent), its inverse exists and equation (2-2) can be written as

$$\underline{Y} = \underline{IV}^{-1} \tag{2-2a}$$

where the choice of the n linearly independent dc n-vector excitation signals is completely arbitrary. For convenience, therefore, let

$$\underline{V} = \underline{U}$$

where $\underline{U}$ is the nxn unit matrix, allowing the nxn admittance matrix $\underline{Y}$ to be completely specified in terms of port current response signals by

$$\underline{Y} = \underline{I} \; . \tag{2-3}$$

The n-port resistance design problem with dependent sources can finally be stated as follows:

> Given a real, constant, nxn admittance matrix $\underline{Y}$, obtain an n-port network comprised exclusively of real, positive-valued resistances and of any combination of the four basic types of dependent sources such that the nxn nonsingular voltage excitation matrix $\underline{V} = \underline{U}$ yields the nxn current response matrix $\underline{I} = \underline{Y}$ (or indicate nonrealizability when appropriate).

B. CHARACTERISTICS OF THE N-PORT NETWORK

1. With Two-Terminal Resistance Elements

The resistance n-port network contains a maximum of 2n nodes and a minimum of n+1 nodes where internal nodes have been suppressed with the star-mesh transformations [4]. The minimal node configuration occurs when there is a single node common to all ports. The maximum number of internal two-terminal resistance branches, defined to be n_g-n, is

$$n_g - n = (2n-1) + (2n-2) + \ . \ . \ . + 1$$
$$= n(2n) - n = n(2n-1) \ .$$

Addition of an independent voltage source at each port brings the total number of branches to $n_g = n(2n-1)+n = 2n^2$. In general, the total maximum number of two-terminal branches, n_g-n, connecting n_n nodes is

$$n_g - n = (n_n)!/2!(n_n-2)! = n_n(n_n-1)/2.$$

The minimal branch, minimal node configuration is found to be

$$n_g - n = ((n+1)-1) + ((n+1)-2) + \ . \ . \ . + 1$$
$$= n(n+1) - n = n^2 - n \ .$$

Again, addition of an independent voltage source at each port brings the total number of branches to $n^2-n+n = n^2$.

Summarizing, the general n-port resistance network contains n_n nodes, where $(n+1 \leq n_n \leq 2n)$, and n_g total internal plus excitation branches, where $(n^2 \leq n_g \leq 2n^2)$.

2. <u>With Two-Terminal Resistance Elements and Dependent Sources</u>

The significance of the addition of dependent sources in the problem formulation lies in the fact that more than one branch may now connect a given pair of nodes in the network. Moreover, each node-pair which is connected by a single two-terminal resistance element is allowed the influence of every other similar node-pair in the network by using the appropriate dependent sources.

A typical pair of nodes is shown in Fig. 2.2a which illustrates the introduction of dependent sources into an n-port resistance network. Each summation set equal to zero represents a set of D dependent source controlling branches, where D is equal to the number of internal resistance branches in the network. For example, $\sum_D V_C^{IV} = 0$ represents a set of D current-controlled voltage source controlling branches. Similarly, the remaining summations each represent a set of C dependent source dependent branches, where C is also equal to the number of internal resistance branches in the network. For example, $\sum_C V_D^{VV}$ represents a set of C voltage-controlled voltage source dependent branches.

Accounting for each branch in the network is critical in the subsequent derivation of the gradient expression,

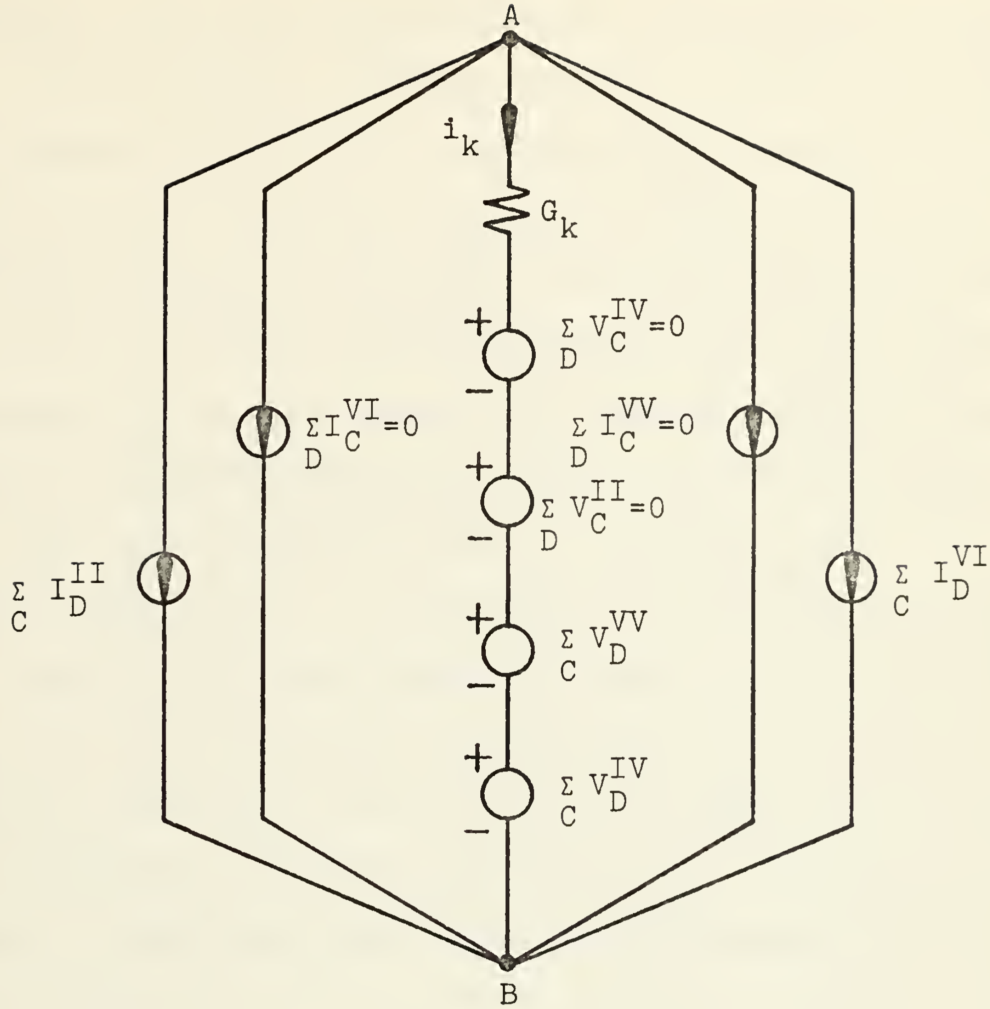

Fig. 2.2a. General node-pair configuration.

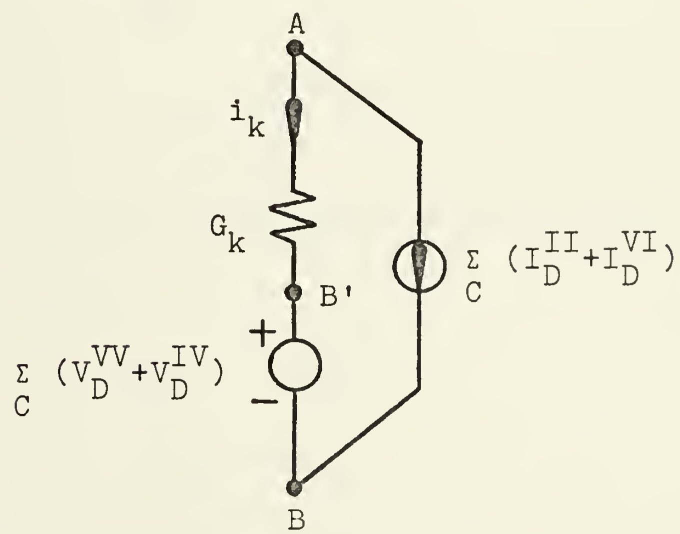

Fig. 2.2b. Simplified node-pair configuration.

therefore a standard notation is now introduced for numbering the network branches. First, the port excitation branches are numbered from 1 to n. The resistance branches are then numbered from n+1 to n_g. Finally the dependent sources are numbered from n_g+1 to n_b, where the controlling branch and the dependent branch of each dependent source are numbered sequentially, in that order. For example, the controlling branch of the first dependent source considered is numbered n_g+1 while the dependent branch is numbered n_g+2.

All that remains in the problem formulation is to specify the limits on the total number of nodes n_n and of branches n_b due to the introduction of dependent sources into the network. Referring to Fig. 2.2a, each summation in series with the branch resistance represents n_g-n possible voltage sources. Since there are four different summations, the maximum number of nodes n_n in the network is found to be

$$n_n = 2n + 4(n_g-n) = 4n_g-2n.$$

Likewise, each of the four different summations connected to node-pair AB represents n_g-n possible current sources. The maximum total number of internal plus excitation branches is found to be

$$n_b = n_g + 4(n_g-n) = 5n_g-4n.$$

To simplify the topological analysis of the network, only the nonzero branches of Fig. 2.2a need be considered, reducing the maximum number of nodes and branches required

to define the network. Accordingly, Fig. 2.2b illustrates the application of superposition to the general node-pair configuration of Fig. 2.2a. The resulting simplified node-pair configuration leads to an equivalent but simplified form of the original network.

Referring to Fig. 2.2b, the presence of one or more voltage-controlled voltage source dependent branches or of one or more current-controlled current source dependent branches creates an additional node (denoted B') for each of the existing n_g-n resistance branches. The maximum number of nodes becomes $2n+(n_g-n) = n_g+n$. Similarly, the presence of one or more current-controlled current source dependent branches or of one or more voltage-controlled current source dependent branches creates an additional branch which also connects none-pair AB. Therefore, the maximum number of resistance branches, simplified voltage source dependent branches, and simplified current source dependent branches, denoted by n_s-n, is

$$n_s-n = n_g-n+2(n_g-n) = 3(n_g-n) .$$

An additional excitation branch at each port brings the total maximum number of branches for the topological analysis to

$$n_s = 3(n_g-n)+n = 3n_g-2n .$$

For convenience in the simplified network, excitation branches are numbered from 1 to n, resistance branches are

numbered from n+1 to n_g, and dependent source dependent branches are numbered from n_g+1 to n_s.

Note that zero-valued voltage and current sources as shown in Fig. 1.1 are indeed legitimate branches in the original unsimplified network. Their inclusion in the problem formulation will become more clear when the adjoint network is formally defined in the next section.

Summarizing, the general n-port resistance network with dependent sources contains n_n nodes where $(n+1) \leq n_n \leq (4n_g-2n)$, and n_b branches where $(n^2) \leq n_b \leq (5n_g-4n)$. The simplified form of the network contains n_n nodes where $(n+1) \leq n_n \leq (n_g+n)$, and n_s branches where $(n^2) \leq n_s \leq (3n_g-2n)$.

Complete specification, therefore, of the given nxn admittance matrix $\underline{Y}$ assumes n separate measurement situations for the n-port resistance network with dependent sources. Each measurement situation claims a set of n dc port voltage excitation signals and n dc port current response signals and an overall set of n_b dc branch voltage and current signals.

III. DERIVATION OF THE DESIGN SCHEME

A. OBTAINING THE GRADIENT EXPRESSION

The assumption has been made that the given nxn admittance matrix may not be realized exactly. In addition, since the admittance matrix is completely specified in terms of network port current response signals by equation (2-3), $\underline{Y} \doteq \underline{I}$, an approximation criterion may be written from equation (1-1) as

$$J(\underline{p}) = \frac{1}{2} \sum_{\ell=1}^{n} \sum_{k=1}^{n} w_{k\ell}(i_{k\ell}-\hat{i}_{k\ell})^2 \qquad (3\text{-}1)$$

where $\underline{p}$ now represents the $4(n_g-n)$-vector of dependent source parameters to be optimized. As before, the $w_{k\ell}$ are the weighting factors, and $\hat{i}_{k\ell}$ is the desired value and $i_{k\ell}$ is the actual value of the k^{th} dc port response current due to the ℓ^{th} linearly independent dc n-vector port voltage excitation.

A relationship is sought between dependent source parameters and the given squared error approximation or performance criterion. This knowledge should allow adjustment of the parameters such that $J(\underline{p})$ can be minimized. Clearly, the gradient of $J(\underline{p})$ with respect to dependent source parameters $\underline{p}$ is the desired relation. Director and Rohrer [2] have outlined the derivation of the gradient expression for the pure resistance n-port network upon

application of Tellegen's theorem [5]. The derivation which follows is based on methods of variational calculus, and it parallels that of Director [3].

Minimization of the performance measure is subject to constraints imposed by network topology and by branch relations. A topological constraint with respect to Kirchhoff's current law is written as

$$\sum_{k=1}^{n_b} q_{mk} i_k = 0 \qquad m = 1,2,\ldots,n_n-1$$

where $\underline{Q} = [q_{mk}]$ is any n_n-1 by n_b fundamental cutset matrix, and i_k is the k^{th} branch current. Kirchhoff's voltage law specifies a second topological constraint which is written as

$$\sum_{k=1}^{n_b} b_{mk} v_k = 0 \qquad m = 1,2,\ldots,n_b-n_n+1$$

where $\underline{B} = [b_{mk}]$ is any n_b-n_n+1 by n_b fundamental loop matrix and v_k is the k^{th} branch voltage.

Branch relations for the network elements are written as

$v_k-\hat{v}_k = 0$	for k^{th} independent voltage source, where $\hat{v}_k$ is the desired value and v_k is the actual value;
$G_k v_k - i_k = 0$	for k^{th} conductance branch;

$$I_C^{VV} = 0$$

$$V_D^{VV} - \mu V_C^{VV} = 0$$

for voltage-controlled voltage sources;

$$I_C^{VI} = 0$$

$$I_D^{VI} - g_m V_C^{VI} = 0$$

for voltage-controlled current sources;

$$V_C^{IV} = 0$$

$$V_D^{IV} - r_m I_C^{IV} = 0$$

for current-controlled voltage sources;

$$V_C^{II} = 0$$

$$I_D^{II} - \beta I_C^{II} = 0$$

for current-controlled current sources.

Consistent with the notation in Fig. 1.1, the C and D subscripts indicate controlling and dependent branches, respectively, for the dependent sources.

The topological constraints and branch relations are now appended to the performance measure using Lagrange multipliers. The augmented performance measure is then written as

$$J(\underline{p}) = \sum_{\ell=1}^{n} \{ \frac{1}{2} \sum_{k=1}^{n} w_{k\ell}(i_{k\ell}-\hat{i}_{k\ell})^2$$

$$+ \sum_{m=1}^{n_n-1} \lambda^t_{m\ell} \sum_{k=1}^{n_b} q_{mk} i_{k\ell} + \sum_{m=1}^{n_b-n_n+1} \lambda^\ell_{m\ell} \sum_{k=1}^{n_b} b_{mk} v_{k\ell}$$

$$+ \sum_{k=1}^{n} \lambda^V_{k\ell}(v_{k\ell}-\hat{v}_{k\ell}) + \sum_{k=n+1}^{n_g} \lambda^G_{k\ell}(G_k v_{k\ell}-i_{k\ell})$$

$$+ \sum_{k=n_g+1}^{n_b} [\lambda^{VV}_{k\ell 1} I^{VV}_{k\ell} + \lambda^{VV}_{k\ell 2} (V^{VV}_{k\ell}-\mu_k V^{VV}_{k\ell})$$

$$+ \lambda^{VI}_{k\ell 1} I^{VI}_{k\ell} + \lambda^{VI}_{k\ell 2} (I^{VI}_{k\ell}-gm_k V^{VI}_{k\ell})$$

$$+ \lambda^{IV}_{k\ell 1} V^{IV}_{k\ell} + \lambda^{IV}_{k\ell 2} (V^{IV}_{k\ell}-rm_k I^{IV}_{k\ell})$$

$$+ \lambda^{II}_{k\ell 1} V^{II}_{k\ell} + \lambda^{II}_{k\ell 2} (I^{II}_{k\ell}-\beta_k I^{II}_{k\ell})] \} . \qquad (3\text{-}2)$$

Although the network parameters doe not appear explicitly in the original performance measure (3-1), their influence is revealed in terms of the port current response $i_k(k=1, 2, \ldots, n)$. Hence, if a given network parameter is incrementally varied

$$p \rightarrow p + \delta p$$

then

$$i_k \rightarrow i_k + \delta i_k$$

and

$$v_k \rightarrow v_k + \delta v_k.$$

Upon introducing an incremental variation in all dependent source parameters, branch relations for the network elements become

$(v_k+\delta v_k) - \hat{v}_k = 0$ for k^{th} independent voltage source;

$G_k(v_k+\delta v_k) - (i_k+\delta i_k) = 0$ for k^{th} conductance branch;

$I_C^{VV} + \delta I_C^{VV} = 0$ for voltage-controlled voltage sources

$$(V_D^{VV}+\delta V_D^{VV}) - (\mu+\delta\mu)(V_C^{VV}+\delta V_C^{VV}) = 0$$

$I_C^{VI} + \delta I_C^{VI} = 0$ for voltage-controlled current sources;

$$(I_D^{VI}+\delta I_D^{VI}) - (g_m+\delta g_m)(V_C^{VI}+\delta V_C^{VI}) = 0$$

$V_C^{IV}+\delta V_C^{IV} = 0$ for current-controlled voltage sources;

$$(V_D^{IV}+\delta V_D^{IV}) - (r_m+\delta r_m)(I_C^{IV}+\delta I_C^{IV}) = 0$$

$$V_C^{II}+\delta V_C^{II} = 0 \qquad \text{for current-controlled current sources.}$$

$$(I_D^{II}+\delta I_D^{II}) - (\beta+\delta\beta)(I_C^{II}+\delta I_C^{II}) = 0$$

Note that although branch conductances are held constant, variations of conductance branch voltage and current appear due to the introduction of the dependent sources.

Having incremented all dependent source parameters, equation (3-2) can be rewritten as

$$J(\underline{p}+\delta\underline{p}) = \sum_{\ell=1}^{n} \{ \frac{1}{2} \sum_{k=1}^{n} w_{k\ell}((i_{k\ell}+\delta i_{k\ell})-\hat{i}_{k\ell})^2$$

$$+ \sum_{m=1}^{n_n-1} \lambda_{m\ell}^{t} \sum_{k=1}^{n_b} q_{mk}(i_{k\ell}+\delta i_{k\ell}) + \sum_{m=1}^{n_b-n_n+1} \lambda_{m\ell}^{\ell} \sum_{k=1}^{n_b} b_{mk}(v_{k\ell}+\delta v_{k\ell})$$

$$+ \sum_{k=1}^{n} \lambda_{k\ell}^{V}(v_{k\ell}+\delta v_{k\ell}-\hat{v}_{k\ell}) + \sum_{k=n+1}^{n_g} \lambda_{k\ell}^{G}(G_k(v_{k\ell}+\delta v_{k\ell})-(i_{k\ell}+\delta i_{k\ell}))$$

$$+ \sum_{k=n_g+1}^{n_b} [\lambda_{k\ell 1}^{VV}(I_{k\ell}^{VV}+\delta I_{k\ell}^{VV}) + \lambda_{k\ell 2}^{VV}((V_{k\ell}^{VV}+\delta V_{k\ell}^{VV}) - (\mu_k+\delta\mu_k)(V_{k\ell}^{VV}+\delta V_{k\ell}^{VV}))$$

$$+ \lambda_{k\ell 1}^{VI}(I_{k\ell}^{VI}+\delta I_{k\ell}^{VI}) + \lambda_{k\ell 2}^{VI}((I_{k\ell}^{VI}+\delta I_{k\ell}^{VI}) - (gm_k+\delta gm_k)(V_{k\ell}^{VI}+\delta V_{k\ell}^{VI}))$$

$$+ \lambda_{k\ell 1}^{IV}(V_{k\ell}^{IV}+\delta V_{k\ell}^{IV}) + \lambda_{k\ell 2}^{IV}((V_{k\ell}^{IV}+\delta V_{k\ell}^{IV}) - (rm_k+\delta rm_k)(I_{k\ell}^{IV}+\delta I_{k\ell}^{IV}))$$

$$+ \lambda_{k\ell 1}^{II}(V_{k\ell}^{II}+\delta V_{k\ell}^{II}) + \lambda_{k\ell 2}^{II}((I_{k\ell}^{II}+\delta I_{k\ell}^{II}) - (\beta_k+\delta\beta_k)(I_{k\ell}^{II}+\delta I_{k\ell}^{II}))\ ]\} . \tag{3-3}$$

Subtracting (3-2) from (3-3) and neglecting second order terms leaves the first order variation of the augmented performance measure written as

$$\delta J(\underline{p}) = J(\underline{p}+\delta\underline{p}) - J(\underline{p})$$

$$= \sum_{\ell=1}^{n} \{ \sum_{k=1}^{n} w_{k\ell}(i_{k\ell}-\hat{i}_{k\ell})\delta i_{k\ell}$$

$$+ \sum_{m=1}^{n_n-1} \lambda^{t}_{m\ell} \sum_{k=1}^{n_b} q_{mk}\delta i_{k\ell} + \sum_{m=1}^{n_b-n_n+1} \lambda^{\ell}_{m\ell} \sum_{k=1}^{n_b} b_{mk}\delta v_{k\ell}$$

$$+ \sum_{k=1}^{n} \lambda^{V}_{k\ell}\delta v_{k\ell} + \sum_{k=n+1}^{n_g} \lambda^{G}_{k\ell}(G_k\delta v_{k\ell}-\delta i_{k\ell})$$

$$+ \sum_{k=n_g+1}^{n_b} [\lambda^{VV}_{k\ell 1}\delta I^{VV}_{k\ell} + \lambda^{VV}_{k\ell 2}(\delta V^{VV}_{k\ell}-\mu_k\delta V^{VV}_{k\ell}-V^{VV}_{k\ell}\delta\mu_k)$$

$$+ \lambda^{VI}_{k\ell 1}\delta I^{VI}_{k\ell} + \lambda^{VI}_{k\ell 2}(\delta I^{VI}_{k\ell}-gm_k\delta V^{VI}_{k\ell}-V^{VI}_{k\ell}\delta gm_k)$$

$$+ \lambda^{IV}_{k\ell 1}\delta V^{IV}_{k\ell} + \lambda^{IV}_{k\ell 2}(\delta V^{IV}_{k\ell}-rm_k\delta I^{IV}_{k\ell}-I^{IV}_{k\ell}\delta rm_k)$$

$$+ \lambda^{II}_{k\ell 1}\delta V^{II}_{k\ell} + \lambda^{II}_{k\ell 2}(\delta I^{II}_{k\ell}-\beta_k\delta I^{II}_{k\ell}-I^{II}_{k\ell}\delta\beta_k)]\}. \qquad (3\text{-}4)$$

After rearranging terms, equation (3-4) can be written in a more useful form as

$$\delta J(\underline{p}) = \sum_{\ell=1}^{n} \left\{ \left| \sum_{m=1}^{n_n-1} \lambda^t_{m\ell} \sum_{k=1}^{n_b} q_{mk} + \left[\sum_{k=1}^{n} w_{k\ell}(i_{k\ell}-\hat{i}_{k\ell}) - \sum_{k=n+1}^{n_g} \lambda^G_{k\ell} \right.\right.\right.$$

$$\left.\left. + \sum_{k=n_g+1}^{n_b} (\lambda^{VV}_{k\ell 1} + \lambda^{VI}_{k\ell 1} + \lambda^{VI}_{k\ell 2} - rm_k \lambda^{IV}_{k\ell 2} + (1-\beta_k)\lambda^{II}_{k\ell 2}) \right] \right| \delta i_{k\ell}$$

$$+ \left| \sum_{m=1}^{n_b-n_n+1} \lambda^{\ell}_{m\ell} \sum_{k=1}^{n_b} b_{mk} + \left[\sum_{k=1}^{n} \lambda^V_{k\ell} + \sum_{k=n+1}^{n_g} \lambda^G_{k\ell} G_k \right.\right.$$

$$\left.\left. + \sum_{k=n_g+1}^{n_b} ((1-\mu_k)\lambda^{VV}_{k\ell 2} - gm_k \lambda^{VI}_{k\ell 2} + \lambda^{IV}_{k\ell 1} + \lambda^{IV}_{k\ell 2} + \lambda^{II}_{k\ell 1}) \right] \right| \delta v_{k\ell}$$

$$\left. - \sum_{k=n_g+1}^{n_b} (\lambda^{VV}_{k\ell 2} V^{VV}_{k\ell} \delta\mu_k + \lambda^{VI}_{k\ell 2} V^{VI}_{k\ell} \delta gm_k + \lambda^{IV}_{k\ell 2} I^{IV}_{k\ell} \delta rm_k + \lambda^{II}_{k\ell 2} I^{II}_{k\ell} \delta\beta_k) \right\} . \tag{3-5}$$

Equation (3-5) has thus been written as a summation of three general terms: (1) terms multiplied by current variation $\delta i_{k\ell}$, (2) terms multiplied by voltage variation $\delta v_{k\ell}$, and (3) terms which include variation of all dependent source parameters. Hence, the desired gradient information lies in the third general term of equation 3-5. The final form of the gradient expression on which the iterative design algorithm is to be based will appear following definition and treatment of the adjoint network.

B. DESCRIPTION OF THE ADJOINT NETWORK

Attention is now given to the description of the so-called adjoint network in order to gain a useful interpretation of the Lagrange multipliers introduced in the augmentation of the performance measure. In addition to the original network $\mathcal{N}$, consider an adjoint network $\tilde{\mathcal{N}}$ which is topologically equivalent. Tree branch voltages and network branch voltages in the adjoint network (as in any electrical network [6]) must be related by

$$[q_{mk}]^T[v_m] - [v_k] = [0] \tag{3-6}$$

where v_m is an $(n_n-1$ by $1)$-vector of tree branch voltages and v_k is an $(n_n$ by $1)$-vector of network branch voltages. Likewise, link currents and network branch currents in the adjoint must be related by

$$[b_{mk}]^T[i_m] - [i_k] = [0] \tag{3-7}$$

where i_m is an $(n_b-n_n+1$ by $1)$-vector of link currents and i_k is an $(n_n$ by $1)$-vector of branch currents. Since the only restriction on the adjoint network $\tilde{\mathcal{N}}$ is that it be topologically equivalent to the original network $\mathcal{N}$, the branches of $\tilde{\mathcal{N}}$ are free to be specified in some convenient manner.

Consider the first general term of equation (3-5). Define λ_k^t as the k^{th} tree branch voltage in $\tilde{\mathcal{N}}$. Now define $- w_k(i_k-\hat{i}_k)$ as the k^{th} port dc voltage excitation and λ_k^G

as the k^{th} conductance branch voltage in $\tilde{\eta}$. Similarly define $-\lambda_{k1}^{VV}$, $-\lambda_{k1}^{VI}$, $-\lambda_{k2}^{VI}$, $rm_k\lambda_{k2}^{IV}$, $-\lambda_{k2}^{II}$, and $\beta_k\lambda_{k2}^{II}$ as the remaining branch voltages in $\tilde{\eta}$. With these definitions, the first general term of equation (3-5) is seen to be identical in form to equation (3-6), and this term vanishes.

Now consider the second general term of equation (3-5). Define λ_k^{ℓ} as the k^{th} link current in $\tilde{\eta}$. Define λ_k^V as the current in the k^{th} port dc voltage excitation branch and $\lambda_k^G G_k$ as the k^{th} conductance branch current in $\tilde{\eta}$. Similarly define λ_{k2}^{VV}, $-\mu_k\lambda_{k2}^{VV}$, $-gm_k\lambda_{k2}^{VI}$, λ_{k1}^{IV}, λ_{k2}^{IV}, and λ_{k1}^{II} as the remaining branch currents in $\tilde{\eta}$. With these definitions, the second general term of equation (3-5) is seen to be identical in form to equation (3-7), and this term likewise vanishes.

Having eliminated the first two terms of equation (3-5), it can now be written as

$$\delta J(\underline{p}) = \sum_{\ell=1}^{n} \sum_{k=n_g+1}^{n_b} -\left(\lambda_{k\ell 2}^{VV} V_{k\ell}^{VV} \delta\mu_k + \delta_{k\ell 2}^{VI} V_k^{VI} \delta gm_k \right.$$

$$\left. + \lambda_{k\ell 2}^{IV} I_{k\ell}^{IV} \delta rm_k + \lambda_{k\ell 2}^{II} I_{k\ell}^{II} \delta\beta_k \right),$$

or using vector notation as

$$\delta J(\underline{p}) = \sum_{\ell=1}^{n} \{ [\nabla J(\underline{p})]^T [\delta \underline{P}] \} , \tag{3-8}$$

where the (n_b-n_g by 1) gradient vector $\nabla J(p)$ is defined as

$$\nabla J(\underline{p}) = \begin{bmatrix} -\lambda_{k2}^{VV} V_k^{VV} \\ \vdots \\ -\lambda_{k2}^{VI} V_k^{VI} \\ \vdots \\ -\lambda_{k2}^{IV} V_k^{IV} \\ \vdots \\ -\lambda_{k2}^{II} I_k^{II} \end{bmatrix}, \tag{3-8a}$$

and the (n_b-n_g by 1) parameter variation vector $\delta\underline{P}$ is defined as

$$\delta\underline{P} = \begin{bmatrix} \delta\mu_k \\ \vdots \\ \delta gm_k \\ \vdots \\ \delta rm_k \\ \vdots \\ \delta\beta_k \end{bmatrix}. \tag{3-8b}$$

The remaining definitions for the adjoint network follow directly from the derivation and subsequent manipulation of equation (3-5). To simplify notation, the ℓ subscript will not be included in defining branch voltages and currents in $\tilde{\eta}$, although it actually exists for each of the n different n-vector excitation situations in the adjoint. Now,

define for $\tilde{\eta}$

$\Psi_k = -w_k(i_k - \hat{i}_k)$	k^{th} port dc voltage excitation branch voltage;
$\Phi_k = \lambda_k^V$	k^{th} port dc voltage excitation branch current;
$\Psi_k^G = \lambda_k^G$	k^{th} conductance branch voltage;
$\Phi_k^G = G_k \lambda_k^G$	k^{th} conductance branch current.

For current-controlled current sources, define

$\Psi_D^{VV} = 0$	controlling branch voltages;
$\Phi_D^{VV} = \lambda_{C2}^{VV}$	controlling branch currents;
$\Phi_C^{VV} = -\mu_D \lambda_D^{VV}$	dependent branch currents;
$\Psi_C^{VV} = -\lambda_{C1}^{VV}$	dependent branch voltages.

For voltage-controlled current sources, define

$\Phi_D^{VI} = 0$	controlling branch currents;
$\Psi_D^{VI} = -\lambda_{C2}^{VI}$	controlling branch voltages;
$\Phi_C^{VI} = gm_D \Psi_D^{VI}$	dependent branch currents;
$\Psi_C^{VI} = -\lambda_{C1}^{VI}$	dependent branch voltages.

For current-controlled voltage sources, define

$\Psi_D^{IV} = 0$	controlling branch voltages

$\Phi_D^{IV}=\lambda_{C2}^{IV}$ controlling branch currents;

$\Psi_C^{IV}=rm_D\Phi_D^{IV}$ dependent branch voltages;

$\Phi_C^{IV}=\lambda_{C1}^{IV}$ dependent branch currents.

For voltage-controlled voltage sources, define

$\Phi_D^{II}=0$ controlling branch currents;

$\Psi_D^{II}=-\lambda_{C2}^{II}$ controlling branch voltages;

$\Psi_C^{II}=-\beta_D\Psi_D^{II}$ dependent branch voltages;

$\Phi_C^{II}=-\lambda_{C1}^{II}$ dependent branch currents.

In summary, the adjoint network $\tilde{\mathcal{N}}$ is related to the original network $\mathcal{N}$ in the following manner.

(i) $\mathcal{N}$ and $\tilde{\mathcal{N}}$ are topologically identical.

(ii) All conductance brances of value G in $\mathcal{N}$ are associated with conductance branches of value G in $\tilde{\mathcal{N}}$.

(iii) Voltage-controlled voltage sources with parameter μ in $\mathcal{N}$ are associated with current-controlled current sources with parameter $-\mu$ in $\tilde{\mathcal{N}}$, and the roles of controlling and dependent branches in $\mathcal{N}$ are reversed in $\tilde{\mathcal{N}}$.

(iv) Current-controlled current sources with parameter β in $\mathcal{N}$ are associated with voltage-controlled voltage sources with parameter $-\beta$ in $\tilde{\mathcal{N}}$, and the roles of controlling and dependent branches in $\mathcal{N}$ are reversed in $\tilde{\mathcal{N}}$.

(v) Voltage-controlled current sources with parameter g_m and current-controlled voltage sources with parameter r_m in η are associated with voltage-controlled current sources with parameter g_m and current-controlled voltage sources with parameter r_m, respectively, in $\tilde{\eta}$ with the roles of controlling and dependent branches in η being reversed in $\tilde{\eta}$.

In conclusion, utilizing previous definitions, equation (3-8a), which is the expression for the gradient of the performance measure with respect to dependent source parameters, can be written in final form as

$$\nabla J(\underline{p}) = \begin{bmatrix} -\ \Phi_D^{VV} \ V_C^{VV} \\ \vdots \\ \Psi_D^{VI} \ V_C^{VI} \\ \vdots \\ -\ \Phi_D^{IV} \ I_C^{IV} \\ \vdots \\ \Psi_D^{II} \ I_C^{II} \end{bmatrix} . \tag{3-9}$$

In order to minimize the performance measure, the dependent source parameters are to be adjusted in the negative gradient direction, $- \nabla J(\underline{p})$. Physically, the gradient information is seen to be products of controlling branch voltages or currents in η and corresponding dependent branch voltages or currents in $\tilde{\eta}$.

C. A NOTATIONAL EXAMPLE

An example will now be given to illustrate notation, to illustrate the correspondence between the original and adjoint networks, and to identify components of the gradient expression.

Figure 3.1 shows a simple 2-port resistance network, $\mathcal{N}$, with 3 different types of dependent sources. Since the example is intended to illustrate notation, numerical values for the resistances and for dependent source parameters are not specified. Branches are numbered in accordance with the convention previously established in problem formulation.

Figure 3.2 shows the related adjoint network, $\tilde{\mathcal{N}}$, generated with the aid of relationships defined in the previous section. Note that the original and adjoint networks are topologically equivalent.

The components of the desired gradient information can now be identified in terms of voltages and currents in $\mathcal{N}$ and in $\tilde{\mathcal{N}}$. The gradient expression, equation (3-9), for this example is

$$\nabla J(\underline{p}) = \begin{bmatrix} -\ \Phi_6^{VV}\ V_6^{VV} \\ \\ -\ \Phi_8^{IV}\ I_8^{IV} \\ \\ \Psi_{10}^{II}\ I_{10}^{II} \end{bmatrix}, \qquad (3\text{-}10)$$

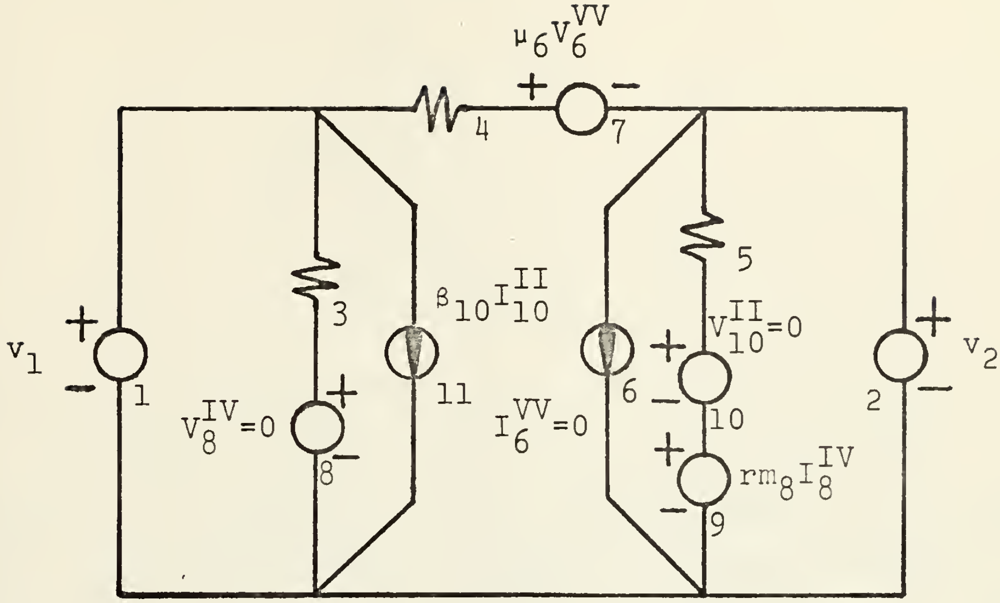

Fig. 3.1. Example 2-port network (original).

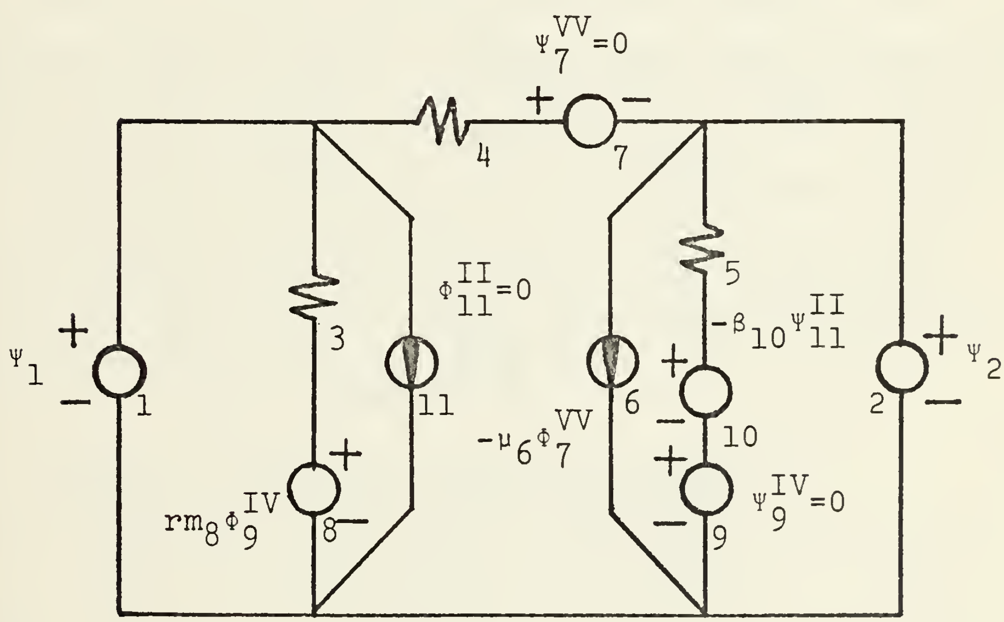

Fig. 3.2. Example 2-port network (adjoint).

and the parameter variation is given by

$$\delta \underline{P} = \begin{bmatrix} \delta\mu_6 \\ \delta rm_8 \\ \delta\beta_{10} \end{bmatrix}.$$

The first term of equation (3-10) is seen to be the negative of the product of the voltage-controlled voltage source controlling branch voltage in $\mathcal{N}$ and the current-controlled current source dependent branch current in $\tilde{\mathcal{N}}$. Similarly, the second term is seen to be the negative of the product of the current-controlled voltage source controlling branch current in $\mathcal{N}$ and the current-controlled voltage source dependent branch current in $\tilde{\mathcal{N}}$. Finally, the third term of equation (3-9) is the product of the current-controlled current source controlling branch current in $\mathcal{N}$ and the voltage-controlled voltage source dependent branch voltage in $\tilde{\mathcal{N}}$.

D. A PROPOSED ITERATIVE DESIGN ALGORITHM

An iterative algorithm for the design of resistance n-port networks with dependent sources is now proposed.

(i) Initially, let all dependent source parameters be equal to zero.

(ii) Apply the ℓ^{th} (ℓ=1, 2, . . ., n) of the n linearly independent n-vector dc voltage excitations to $\mathcal{N}$.

(iii) Measure all dependent source controlling branch voltages and currents in $\mathcal{N}$.

(iv) Calculate the current response errors at the n ports.

(v) Apply these n port current response errors as voltage excitation at the corresponding ports of $\tilde{\mathcal{N}}$.

(vi) Measure all dependent source controlling branch voltages and currents in $\tilde{\mathcal{N}}$ and multiply them with corresponding dependent source controlling branch voltages and currents in $\mathcal{N}$. This forms the ℓ^{th} set of the maximum $4(n_g-n)$ components of the gradient expression.

(vii) Let $\ell = \ell + 1$ and return to step (ii) being certain upon reaching step (vi) to add newly calculated components of the gradient expression to the corresponding previously calculated components. Execute this step for $(\ell=1, 2, \ldots, n)$ thereby obtaining the complete gradient.

(viii) Adjust the dependent source parameters in the negative gradient direction.

(ix) Repeat the entire procedure from step (ii) until the performance measure has been suitably minimized or until some other termination criterion has been satisfied.

A number of comments are in order with regard to this proposed algorithm. Step (ii), for a given ℓ, normally implies inserting a 1 volt source across the $k\ell^{th}$ port where $k=\ell$ and inserting 0 volt sources across the remaining

n-1 ports. Steps (iii) and (vi) assume the availability of a general circuit analysis program such as the IBM ECAP program [7] for measuring voltages and currents in both the original and adjoint networks.

A final comment deals with step (viii). The dependent source parameters are to be adjusted in the direction of steepest descent for each iteration of the design algorithm; that is

$$\underline{p}^{i+1} = \underline{p}^{i} - \alpha^{i} \left[\nabla J(\underline{p}^{i})\right] \qquad i=1, 2, \ldots$$

where superscript i+1 or i indicates the iteration number and α is a real, nonnegative step size determining constant which may vary from iteration to iteration. The proper choice of α is critical using the steepest descent algorithm outlined here and a good discussion regarding the selection of α and the use of the steepest descent technique can be found in [8].

IV. CONCLUSION

The classical n-port resistance design problem has been defined. A practical approach to its solution has been to cast the problem as a signal approximation problem and then to apply parameter optimization theory to minimize an established approximation criterion.

The alternative approach offered involves the introduction of dependent sources into the network holding branch resistance values constant. The dependent source parameters then assumed the primary roles in the parameter optimization problem. Derivation of the gradient expression led to the generation of an iterative design algorithm which could be programmed on the digital computer.

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13. ABSTRACT

The classical n-port resistance network design problem is defined and a common approach to its solution based on parameter optimization is offered. An alternative approach is then proposed in which dependent sources are introduced into the network and where dependent source parameters assume the primary roles in the parameter optimization problem. Development of the alternative approach leads to specification of a related adjoint network and ultimately to the generation of an iterative design algorithm.

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